

Package: PGM2 (via r-universe)

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Type Package

Title Recursive Construction of Nested Resolvable Designs and Associated Uniform Designs over GF(p)

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Description Recursive construction of balanced incomplete block designs (BIBDs), their successive generations, resolvable BIBDs (RBIBDs) and associated uniform designs (UDs), derived from finite projective geometries PG(m, p) over a Galois field GF(p) of any prime order p. Implements and generalises the method of Boudraa, Gheribi-Aoulmi and Laib (2013, International Journal of Research and Reviews in Applied Sciences, 17(2), 167-176), which was previously available only for p = 2, and the uniform design constructions of Fang et al. (2004) [doi:10.1016/S0012-365X\(03\)00100-6](https://doi.org/10.1016/S0012-365X(03)00100-6). Designs of every recursion stage can be extracted, and all constructions are validated against the parameters published in the original paper.

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Encoding UTF-8

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BugReports <https://github.com/mlaib/PGM2/issues>

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BIB

Balanced Incomplete Block Design from PG(m, p)

Description

Builds the symmetric balanced incomplete block design (BIBD) whose treatments are the points of the projective geometry PG(m, p) over the Galois field GF(p) and whose blocks are its hyperplanes.

Usage

BIB(m, p = 2)

Arguments

m Dimension of the projective geometry (an integer, $m \geq 2$).

p Order of the Galois field GF(p); must be prime. Defaults to p = 2, which reproduces the designs of PGM2 ≤ 1.2 exactly.

Details

Treatments are numbered by enumerating the canonical representatives of the projective points in base-p counting order. Block i consists of the points x lying on the hyperplane $a_i \cdot x = 0 \pmod{p}$, where a_i is the i-th point (point-hyperplane duality).

Value

A list with components:

V Number of treatments, $(p^{m+1} - 1)/(p - 1)$.

B Number of blocks (equal to V: the design is symmetric).

R Replication of each treatment, $(p^m - 1)/(p - 1)$.

K Block size (equal to R).

Lambda Concurrence parameter, $(p^{m-1} - 1)/(p - 1)$.

BIB The design: a B x K matrix whose rows are the blocks, containing treatment labels.

Author(s)

Mohamed Laib, Abia Boudraa and Zebida Gheribi-Aoulmi

References

A. Boudraa, Z. Gheribi-Aoulmi and M. Laib (2013). Recursive method for construction of resolvable nested designs and uniform designs associated. *International Journal of Research and Reviews in Applied Sciences*, 17(2), 167–176.

D. Dugue (1958). *Traite de statistique theorique et appliquee*. Masson et Cie, Paris.

Examples

```
X <- BIB(4)          # BIBD (31, 15, 7) from PG(4, 2)
X$V; X$K; X$Lambda
```

```
Y <- BIB(2, p = 3) # BIBD (13, 4, 1): the projective plane of order 3
Y$BIB
```

Gen

BIBD of the Next Generation

Description

Extracts the balanced incomplete block design of the next generation by deleting block n of a BIBD built from a projective geometry: each remaining block is intersected with the deleted one, so the deleted block plays the role of a lower-dimensional projective geometry and the intersections are its hyperplanes.

Usage

```
Gen(n, mat)
```

Arguments

<code>n</code>	Index of the block (sub-variety) to be deleted; an integer between 1 and <code>nrow(mat)</code> .
<code>mat</code>	The matrix of the BIBD (rows are blocks), e.g. the BIB component returned by BIB .

Details

Works for any order p : block sizes are derived from the data instead of the binary-only formula used in `PGM2 <= 1.2` (for a symmetric BIBD every remaining block meets the deleted block in exactly Λ treatments). For a BIBD from $PG(m, p)$, the result is the BIBD of $PG(m - 1, p)$ on the treatments of the deleted block.

Value

A list with components:

- V Number of treatments of the new BIBD.
- B Number of blocks.
- R Replication of each treatment.
- K Size of each block.
- BIB2 The design: a matrix whose rows are the blocks.

Author(s)

Mohamed Laib, Abia Boudraa and Zebida Gheribi-Aoulmi

References

A. Boudraa, Z. Gheribi-Aoulmi and M. Laib (2013). Recursive method for construction of resolvable nested designs and uniform designs associated. *International Journal of Research and Reviews in Applied Sciences*, 17(2), 167–176.

Examples

```
X <- BIB(4)           # BIBD (31, 15, 7) from PG(4, 2)
X2 <- Gen(1, X$BIB)   # second generation: BIBD (15, 7, 3)
X2$V; X2$K

Y <- BIB(3, p = 3)    # BIBD (40, 13, 4) from PG(3, 3)
Y2 <- Gen(1, Y$BIB)   # second generation: BIBD (13, 4, 1)
Y2$V; Y2$K
```

Qn

Reduced Resolvable Design of Stage n and its Uniform Design

Description

Builds the reduced resolvable design Q_n^* of Boudraa et al. (2013) directly, together with its associated uniform design $U(p^m, (p^n)^{r_n^{**}})$: the treatments are the p^m points of the affine geometry AG(m, p), each factor corresponds to one $(m - n)$ -dimensional linear subspace W of $GF(p)^m$ (one parallel class of affine flats), and the level of a treatment x on factor W is the coset $x + W$.

Usage

```
Qn(m, n, p = 2)
```

Arguments

m	Dimension of the projective geometry (an integer, $m \geq 2$).
n	Stage of the recursion, an integer with $1 \leq n \leq m - 1$.
p	Order of the Galois field GF(p); must be prime. Defaults to p = 2.

Details

The number of factors is the Gaussian binomial coefficient $\binom{m}{m-n}_p$. Any two distinct runs coincide in exactly $\binom{m-1}{m-n-1}_p$ factors (the number of $(m-n)$ -dimensional subspaces containing a fixed nonzero vector), so the design is equidistant; consequently it attains the discrete-discrepancy lower bound of Fang et al. (2004). Across stages the levels refine: if $W' \subset W$ then the level partition induced by W is a coarsening of the one induced by W' .

For $n = 1$ the design is the saturated Rao-Hamming orthogonal array $OA(p^m, (p^m - 1)/(p - 1), p, 2)$. For $p = 2, n = 1$, recoding its two levels to ± 1 gives the Sylvester-Hadamard member of the Plackett-Burman class of order 2^m , up to row, column and level equivalence.

Subspaces are enumerated by canonical reduced row echelon form, which generates each subspace exactly once rather than scanning subsets of the $p^m - 1$ nonzero vectors. Labelling then visits, for every subspace and every run, the p^{m-n} elements of a coset, so the cost is approximately $O\left(\binom{m}{m-n}_p p^m p^{m-n}\right)$ plus factor-construction overhead, with output size $O\left(\binom{m}{m-n}_p p^m\right)$. This is why stages with similar factor counts can differ in runtime. On the machine used for the package benchmarks, Qn(5, 2) completed in well under a second, Qn(7, 1) in a few seconds and Qn(7, 3) (11811 factors) in a few minutes; timings are hardware dependent.

Value

A list with components:

V Number of treatments (runs), p^m .

B Number of blocks of Q_n^* .

R Replication of each treatment (= number of factors).

K Block size, p^{m-n} .

Lambda Concurrence parameter (constant, see Details).

Levels Number of levels of each factor, p^n .

UD The uniform design: a $p^m \times R$ matrix of levels 1..Levels.

Author(s)

Mohamed Laib, Abba Boudraa and Zebida Gheribi-Aoulmi

References

- A. Boudraa, Z. Gheribi-Aoulmi and M. Laib (2013). Recursive method for construction of resolvable nested designs and uniform designs associated. *International Journal of Research and Reviews in Applied Sciences*, 17(2), 167–176.
- K.T. Fang, X. Lu, Y. Tang and J. Yin (2004). Constructions of uniform designs by using resolvable packings and coverings. *Discrete Mathematics*, 274, 25–40.

Examples

```

Q <- Qn(3, 1)          # stage 1 of PG(3,2): U(8, 2^7), Plackett-Burman
Q$UD

Q2 <- Qn(3, 2)          # stage 2: U(8, (2^2)^7), Example 3 of the paper
Q32 <- Qn(3, 1, p = 3)  # U(27, 3^13)
c(Q32$V, Q32$B, Q32$R, Q32$K, Q32$Lambda)

Qn(4, 2)$R              # 35 four-level factors on 16 runs

```

Resolvable

Resolvable Balanced Incomplete Block Design

Description

Extracts the resolvable balanced incomplete block design (RBIBD) residual to block n of a BIBD constructed from a projective geometry: block n is deleted together with all its treatments, and the surviving parts of the remaining blocks form a resolvable design (for a BIBD from $PG(m, p)$ this is the design of the affine geometry $AG(m, p)$).

Usage

```
Resolvable(n, mat)
```

Arguments

n Index of the block (sub-variety) to be deleted; an integer between 1 and `nrow(mat)`.

mat The matrix of the BIBD (rows are blocks), e.g. the BIB component returned by [BIB](#).

Details

Works for any order p : residual block sizes are derived from the data instead of the binary-only formula used in `PGM2 <= 1.2` (every remaining block loses exactly Lambda treatments to the deleted block, so all residual blocks have size $K - \text{Lambda}$). The blocks partition into R parallel classes of p blocks each.

Value

A list with components:

V Number of treatments of the RBIBD, p^m .

B Number of blocks.

R Replication of each treatment.

K Size of each block, p^{m-1} .

RBIB The design: a matrix whose rows are the blocks.

Author(s)

Mohamed Laib, Abia Boudraa and Zebida Gheribi-Aoulmi

References

A. Boudraa, Z. Gheribi-Aoulmi and M. Laib (2013). Recursive method for construction of resolvable nested designs and uniform designs associated. *International Journal of Research and Reviews in Applied Sciences*, 17(2), 167–176.

R.C. Bose (1942). A note on the resolvability of balanced incomplete block designs. *Sankhya*, 6, 105–110.

Examples

```
X <- BIB(4)                # BIBD (31, 15, 7) from PG(4, 2)
Y <- Resolvable(1, X$BIB)  # RBIBD (16, 30, 15, 8, 7)
Y$V; Y$B; Y$K

Z <- BIB(2, p = 3)         # BIBD (13, 4, 1) from PG(2, 3)
W <- Resolvable(1, Z$BIB)  # RBIBD (9, 12, 4, 3, 1): AG(2, 3)
W$RBIB
```

Steps	<i>All Stages of the Recursive Construction</i>
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Description

Runs the whole recursion of Boudraa et al. (2013) starting from the projective geometry PG(m, p): at each stage the current BIBD yields a resolvable design (via [Resolvable](#)) and its associated uniform design (via [Uniform](#)), and the next-generation BIBD is extracted (via [Gen](#)) until the geometry is exhausted.

Usage

```
Steps(m, n, stage = "all", p = 2)
```

Arguments

m	Dimension of the projective geometry (an integer, m >= 2).
n	Index of the block (sub-variety) to be deleted at each stage. The same index is used at every stage, and the number of blocks shrinks along the recursion, so n must not exceed the block count of the last stage, $p^2 + p + 1$ (the design of PG(2, p)).
stage	Stages wanted, a character vector (default "all"): 'S1' The first-generation BIBD. 'S2' The BIBDs of the following generations. 'S3' The resolvable designs of every stage.

'S4'

The uniform designs associated with every stage.

p

Order of the Galois field GF(p); must be prime. Defaults to p = 2, which reproduces the designs of PGM2 <= 1.2.

Value

A named list with (depending on stage) components BIB1 (the first-generation BIBD as returned by [BIB](#)), BIBg (a list of the next-generation BIBDs), Resolvables (a list of the resolvable designs of every stage) and UDs (a list of the associated uniform designs).

Note

The return structure is tidier than in PGM2 <= 1.2, where the stages were spliced into one flat, partially unnamed list: each stage is now a proper sub-list.

Author(s)

Mohamed Laib, Abla Boudraa and Zebida Gheribi-Aoulmi

References

A. Boudraa, Z. Gheribi-Aoulmi and M. Laib (2013). Recursive method for construction of resolvable nested designs and uniform designs associated. *International Journal of Research and Reviews in Applied Sciences*, 17(2), 167–176.

Z. Gheribi-Aoulmi and M. Bousseboua (2005). Recursive methods for construction of balanced n-ary block designs. *Serdica Mathematical Journal*, 31, 189–200.

Examples

```
s <- Steps(3, 1)           # all stages of PG(3, 2)
names(s)
s$UDs[[1]]$UD             # U(8, 2^7)

s3 <- Steps(3, 1, p = 3)   # all stages of PG(3, 3)
s3$UDs[[1]]$UD            # U(27, 3^13)

Steps(4, 1, c('S1', 'S4')) # first and last stage only, PG(4, 2)
```

Uniform	<i>Uniform Design from a Resolvable BIBD</i>
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Description

Builds a symmetric uniform design (UD) from a resolvable balanced incomplete block design, following the correspondence of Fang et al.: the blocks are partitioned into parallel classes, each class becomes a factor (column), and the level of a treatment on a factor is the index of the block of that class containing it.

Usage

```
Uniform(mat)
```

Arguments

mat The matrix of the RBIBD (rows are blocks), e.g. the RBIB component returned by [Resolvable](#).

Details

The resolution is found by exact-cover search with backtracking, so the result does not depend on the order of the rows of **mat**: any row permutation yields the same design up to a permutation of the factors and a relabelling of levels. Earlier versions used a first-fit greedy extraction, which could fail on a valid resolvable design presented in an unfavourable row order.

If **mat** admits no resolution, or the search budget is exhausted, an informative error is raised rather than a partial design returned.

Works for any order p (the parallel-class extraction is purely combinatorial). For the RBIBD residual to a block of the BIBD of PG(m , p), the result is a uniform design with p^m runs and p levels per factor.

Value

A list with components:

n Number of experiments (runs) of the uniform design.

F Number of factors (one per parallel class).

UD The design: an $n \times F$ matrix of levels.

classes A list giving, for each factor, the row indices of **mat** forming that parallel class.

Author(s)

Mohamed Laib, Abba Boudraa and Zebida Gheribi-Aoulmi

References

K.T. Fang, X. Lu, Y. Tang and J. Yin (2004). Constructions of uniform designs by using resolvable packings and coverings. *Discrete Mathematics*, 274, 25–40.

K.T. Fang, G.N. Ge, M.Q. Liu and H. Qin (2004). Construction of uniform designs via super-simple resolvable t -designs. *Utilitas Mathematica*, 66, 15–32.

Examples

```
X <- BIB(3)                      # BIBD (15, 7, 3) from PG(3, 2)
Y <- Resolvable(1, X$BIB)       # RBIBD (8, 14, 7, 4, 3)
Uniform(Y$RBIB)$UD            # U(8, 2^7)

Z <- BIB(2, p = 3)              # BIBD (13, 4, 1) from PG(2, 3)
W <- Resolvable(1, Z$BIB)       # RBIBD (9, 12, 4, 3, 1)
```

```
Uniform(W$RBIB)$UD          # U(9, 3^4)

# the result does not depend on the order of the blocks
set.seed(1)
P <- Y$RBIB[sample(nrow(Y$RBIB)), ]
dim(Uniform(P)$UD)
```

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